Harmonic plane waves in a chiral slab

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We illustrate the three-dimensional complex formalism of electromagnetism previously developed [P. Hillion, Phys. Rev. E **47**, 1365 (1993)] for time-harmonic plane waves propagating in an isotropic homogeneous chiral slab.

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I. INTRODUCTION

We discuss in this work the propagation of an electromagnetic plane wave through a homogeneous isotropic chiral slab immersed in a homogeneous isotropic achiral medium. We use the three-dimensional (3D) complex formalism previously developed [1], except that instead of the Tellegen constitutive relations

$$\mathbf{D}=\epsilon\mathbf{E}+\alpha\mathbf{H}\ ,\quad \mathbf{B}=\mu\mathbf{H}+\beta\mathbf{E} \tag{1}$$

we use the Post constitutive relations [2]

$$\mathbf{D}=\epsilon\mathbf{E}+i\gamma\mathbf{B}\ ,\quad \mathbf{H}=i\gamma E+\mu^{-1}B\ ,\quad i=\sqrt{-1} \tag{2}$$

which, in addition to being covariant, make calculations easier and from (1) and (2) we get

$$\alpha=i\gamma\mu\ ,\quad \beta=-i\gamma\mu\ . \tag{2$'$}$$

In these relations $\mathbf{E},\mathbf{H}$ are the electric and magnetic fields, $\mathbf{D}$ the displacement field, B the magnetic induction, ϵ,μ the permittivity and permeability, and α,β,γ the chirality parameters.

In the 3D complex formalism the electromagnetic field is described by a complex vector

$$\mathbf{\Lambda}=\sqrt{\epsilon}\mathbf{E}+i\sqrt{\mu}\mathbf{H}\ , \tag{3}$$

satisfying Maxwell's equations

$$\nabla\times\mathbf{\Lambda}=in^{\pm}\partial_{x_0}\mathbf{\Lambda},\quad \nabla\cdot\mathbf{\Lambda}=0\ ,\quad x_0=ct\ , \tag{4}$$

∇ is the nabla symbol and the refractive index $n^{\pm}$ is

$$n^{\pm}=(\epsilon\mu)^{1/2}(1\pm\xi\gamma)\ ,\quad \xi=\left[\frac{\mu}{\epsilon}\right]^{1/2}\ . \tag{5}$$

We note $\mathbf{\Lambda}^{+}$ (respectively $\mathbf{\Lambda}^{-}$) the solutions of Eqs. (4) with n^{+} (respectively n^{-}). For an achiral medium where $\gamma=0$ we use the notations

$$\epsilon',\ \mu',\ n'=\sqrt{\epsilon'\mu'},\ \xi'=\left[\frac{\mu'}{\epsilon'}\right]^{1/2}\ ,\ \Lambda'\ . \tag{6}$$

Let us now consider the plane-wave solutions of Eqs. (4) when the (x,z) plane is the incidence plane. In an achiral medium, the components Λ'_j $(j=1,2,3)$ of a linearly polarized wave are [1]

$$\Lambda'_j(x,z,x_0)=e^{-ik_0(x_0+n'x\cos\theta+n'z\sin\theta)}F_j(\theta)\psi'\ , \tag{7}$$

with

$$F_1(\theta)=-i\sin\theta\ ,\quad F_2(\theta)=1\ ,\quad F_3(\theta)=i\cos\theta\ , \tag{8}$$

where ψ' is the wave amplitude with components ψ'_E and ψ'_H that we write in agreement with (3):

$$\psi'=\sqrt{\epsilon'}\psi'_E+i\sqrt{\mu'}\psi'_H\ , \tag{9}$$

Two circularly polarized waves propagate in the chiral medium. One is right handed, the other is left handed corresponding to both signs of n in Eqs. (4). The components $\Lambda_j^{\pm}$ of these waves are [1]

$$\Lambda_j^{\pm}(x,z,x_0)=e^{-ik_0(x_0+n^{\pm}x\cos\theta^{\pm}+n^{\pm}z\sin\theta^{\pm})}F_j^{\pm}(\theta^{\pm})\psi^{\pm}\ , \tag{10}$$

with

$$F_1^{+}(\theta^{+})=-i\sin\theta^{+},\quad F_2^{+}(\theta^{+})=1,\quad F_3^{+}(\theta^{+})=i\cos\theta^{+}\ ,$$
$$F_1^{-}(\theta^{-})=i\sin\theta^{-},\quad F_2^{-}(\theta^{-})=1,\quad F_3^{-}(\theta^{-})=-i\cos\theta^{-}\ . \tag{11}$$

But for circularly polarized waves we have $\pm i\sqrt{\mu}H=\sqrt{\epsilon}E$, so we define the amplitudes in (10) by the relations

$$\psi^{+}=2\sqrt{\epsilon}\psi_E\ ,\quad \psi^{-}=-2i\sqrt{\mu}\psi_H\ . \tag{12}$$

For a plane wave incident from an achiral medium on the boundary S of a chiral medium located at $x=0$ the continuity of the phase $e^{ik(\cdots)}$ on S implies the Descartes-Snell relations

$$n'\sin\theta_i=n'\sin\theta_r=n^{+}\sin\theta_t^{+}=n^{-}\sin\theta_t^{-}\ , \tag{13}$$

where θ_i, θ_r, $\theta_t^{\pm}$ are the angles of incidence, reflection, and refraction, respectively.

Now if the incident wave comes from the chiral medium and if the boundary S is located at $x=d$ the Descartes-Snell relations become

$$n^{\pm}\sin\varphi_i^{\pm}=n^{\pm}\sin\varphi_r^{\pm}=n'\sin\varphi_t\ , \tag{14}$$

where $\varphi_i^{\pm}$, $\varphi_r^{\pm}$, φ_t are now the angles of incidence, reflection, and refraction at $x=d$.

Here we want to calculate the amplitudes of the fields reflected and refracted by a chiral slab located between $x=0$ and $x=d$. Of course this requires us to know the boundary conditions at both interfaces.

II. BOUNDARY CONDITIONS

A. Achiral-chiral interface

A linearly polarized wave is incident on the boundary surface S of a chiral medium at $x=0$. In the chiral medium propagate two circularly polarized waves and the boundary conditions requiring the continuity of the tangential components of **E** and **H** across the interface S take the form [1]

$$\sqrt{\epsilon}(\mathrm{Re}\mathbf{\Lambda}')_{y,z}=\sqrt{\epsilon'}(\mathbf{\Lambda}^{+}+\mathbf{\Lambda}^{-})_{y,z}\ ,$$
$$\sqrt{\mu}(\mathrm{Im}\mathbf{\Lambda}')_{y,z}=\sqrt{\mu'}(\mathbf{\Lambda}^{+}-\mathbf{\Lambda}^{-})_{y,z}\ . \tag{15}$$

Now in agreement with (9) and (12) we write the amplitude ψ' and $\psi^{\pm}$ in (7) and (10):

$$\psi_i'=\sqrt{\epsilon'}Q_E+i\sqrt{\mu'}Q_H \quad \text{(incident wave)}\ ,$$
$$\psi_r'=\sqrt{\epsilon'}Q_E+i\sqrt{\mu'}Q_H \quad \text{(reflected wave)}\ , \tag{16}$$
$$\psi_t^{+}=2\sqrt{\epsilon}T_E\ ,\quad \psi_t^{-}=-2i\sqrt{\mu}T_H \quad \text{(refracted waves)}\ .$$

Leaving aside the exponential term in agreement with (13) we get from (7), (10), and (16) for the components $\Lambda'_{y,z}$ and $\Lambda^{\pm}_{y,z}$

$$\Lambda_y'=\sqrt{\epsilon'}(Q_E+R_E)+\sqrt{\mu'}(Q_H+R_H)\ ,$$
$$\Lambda_z'=i\sqrt{\epsilon'}\cos(\theta_i)(Q_E-R_E)-\sqrt{\mu'}\cos(\theta_i)(Q_H-R_H)\ , \tag{17a}$$

where in Λ_z' we used the relation $\cos\theta_r=-\cos\theta_i$ and

$$\Lambda_y^{+}=2\sqrt{\epsilon}T_E\ ,\quad \Lambda_y^{-}=-2i\sqrt{\mu}T_H\ ,$$
$$\Lambda_z^{+}=2i\sqrt{\epsilon}\cos(\theta_t^{+})T_E\ ,\quad \Lambda_z^{-}=-2\sqrt{\mu}\cos(\theta_t^{-})T_H\ . \tag{17b}$$

Substituting (17a) and (17b) into (15) gives the relations

$$Q_H+R_H=T_H-i\xi^{-1}T_E\ ,$$
$$Q_E+R_E=T_E-i\xi T_H\ , \tag{18a}$$

and

$$\xi'\cos(\theta_i)(Q_H-R_H)=\xi\cos(\theta_t^{-})T_H-i\cos(\theta_t^{+})T_E\ , \tag{18b}$$

$$\xi'^{-1}\cos(\theta_i)(Q_E-R_E)=\xi^{-1}\cos(\theta_t^{+})T_E-i\cos(\theta_t^{-})T_H\ .$$

In terms of the incident amplitudes Q_E,Q_H the solution of (18a) and (18b) has the form

$$T_E=T_{11}Q_E+T_{12}Q_H\ ,\quad R_E=R_{11}Q_E+R_{12}Q_H\ ,$$
$$T_H=T_{21}Q_E+T_{22}Q_H\ ,\quad R_H=R_{21}Q_E+R_{22}Q_H\ . \tag{19}$$

Then, introducing the notations

$$a_{11}=\frac{1}{2}\left[1+\frac{\xi}{\xi'}\frac{\cos\theta_t^{-}}{\cos\theta_i}\right]\ ,\quad a_{12}=\frac{-i}{2\xi}\left[1+\frac{\xi}{\xi'}\frac{\cos\theta_t^{+}}{\cos\theta_i}\right]\ , \tag{20a}$$

$$a_{21}=\frac{-i\xi}{2}\left[1+\frac{\xi'}{\xi}\frac{\cos\theta_t^{-}}{\cos\theta_i}\right]\ ,\quad a_{22}=\frac{1}{2}\left[1+\frac{\xi'}{\xi}\frac{\cos\theta_t^{+}}{\cos\theta_i}\right]\ ,$$

and with $A=a_{11}a_{22}-a_{12}a_{21}$,

$$b_{11}=A^{-1}(a_{11}+i\xi a_{12})\ ,\quad b_{12}=A^{-1}(a_{21}+i\xi a_{22})\ , \tag{20b}$$

$$b_{21}=A^{-1}(a_{12}+i\xi^{-i}a_{11})\ ,\quad b_{22}=A^{-1}(a_{22}+i\xi^{-1}a_{21})\ ,$$

we get

$$T_{rs}=A^{-1}a_{rs}\ ,\quad r,s=1,2\ ,$$
$$R_{11}=b_{11}-1,\quad R_{12}=-b_{12}\ , \tag{21}$$
$$R_{21}=-b_{21},\quad R_{22}=b_{22}-1\ .$$

These relations are the Fresnel formulas for a linearly polarized plane waves incident from an achiral medium on the plane boundary of a chiral medium. Similar results have been previously obtained by several authors [3,4,5] using different constitutive relations.

B. Chiral-achiral medium

Let us now consider a couple of right-handed and left-handed circularly polarized plane waves incident from an achiral medium on a plane surface located at $x=d$. It is assumed that these circularly polarized waves were generated by a linearly polarized wave incident on the plane $x=0$.

The boundary conditions (15) are still valid; only the expressions of the wave amplitudes change. Discarding in agreement with (14) the exponentials

$$e^{-ik_0(x_0+nz\sin\varphi)}$$

and introducing the angles

$$\delta_i^{\pm}=k_0n^{\pm}\cos(\varphi_i^{\pm})d=-\delta_r^{\pm}\ ,\quad \delta_t=k_0n'\cos(\varphi_t)d\ , \tag{22}$$

we get from (10), (11), and (12) at $x=d$

$$\Lambda_y^{+}=2\sqrt{\epsilon}(\hat{Q}_Ee^{i\delta_i^{+}}+\hat{R}_Ee^{-i\delta_i^{+}})\ ,$$
$$\Lambda_y^{-}=-2i\sqrt{\mu}(\hat{Q}_He^{i\delta_i^{-}}+\hat{R}_He^{-i\delta_i^{-}})\ ,$$
$$\Lambda_z^{+}=2i\sqrt{\epsilon}\cos\varphi_i^{+}(Q_Ee^{i\delta_i^{+}}-\hat{R}_Ee^{-i\delta_i^{+}})\ , \tag{23}$$
$$\Lambda_z^{-}=-2\sqrt{\mu}\cos\varphi_i^{-}(Q_He^{i\delta_i^{-}}-\hat{R}_He^{-i\delta_i^{-}})\ ,$$

where we used in $\Lambda_z^{\pm}$ the relation $\cos\varphi_t^{\pm}=-\cos\varphi_i^{\pm}$.

Similarly from (7), (8), (9) we have

$$\Lambda_y'=(\sqrt{\epsilon'}\hat{T}_E+i\sqrt{\mu'}\hat{T}_H)e^{i\delta_t}\ ,$$
$$\Lambda_z'=[i\cos(\varphi_t)\sqrt{\epsilon'}\hat{T}_E-\cos(\varphi_t)\sqrt{\mu'}\hat{T}_H]e^{i\delta_t}\ . \tag{24}$$

In (23) and (24) $\hat{Q}$, $\hat{R}$, and $\hat{T}$ represent the amplitudes of the incident reflected and refracted waves at $x=d$. Substituting (23) and (24) into (15) gives the relations

$$\hat{T}_H e^{i\delta_t}=\hat{Q}_H e^{i\delta_i^-}+\hat{R}_H e^{-\delta_i^-}-i\xi^{-1}(\hat{Q}_E e^{i\delta_i^+}+\hat{R}_E e^{-i\delta_i^+}) , \tag{25a}$$

$$\hat{T}_E e^{i\delta_t}=\hat{Q}_E e^{i\delta_i^+}+\hat{R}_E e^{i\delta_i^-}-i\xi(\hat{Q}_H e^{i\delta_i^-}+\hat{R}_H e^{-i\delta_i^-})$$

and

$$\xi' \cos(\varphi_t) e^{i\delta_t}\hat{T}_H=\xi\cos(\varphi_t^-)(\hat{Q}_H e^{i\delta_i^-}-\hat{R}_H e^{-i\delta_i^-}) \\ -i\cos(\varphi_i^+)(\hat{Q}_E e^{i\delta_i^+}-\hat{R}_E e^{-i\delta_i^+}) , \tag{25b}$$

$$\xi'^{-1}\cos(\varphi_t)e^{i\delta_t}\hat{T}_E=\xi^{-1}\cos(\varphi_i^+)(\hat{Q}_E e^{i\delta_i^+}-\hat{R}_E e^{-i\delta_i^+}) \\ -i\cos(\varphi_i^-)(\hat{Q}_H e^{i\delta_i^-}-\hat{R}_H e^{-i\delta_i^-}) .$$

To solve (25a) and (25b) we introduce the functions

$$\alpha^{\pm}(\varphi_i)=1\mp\frac{\xi}{\xi'}\frac{\cos\varphi_i}{\cos\varphi_t} , \quad \beta^{\pm}(\varphi_i)=1\mp\frac{\xi'}{\xi}\frac{\cos\varphi_i}{\cos\varphi_t} , \tag{26}$$

and the matrices $C^{\pm}$ with the elements

$$\begin{aligned} c_{11}^{\pm}&=\pm\xi'\cos(\varphi_t)e^{\pm i\delta_i^-}\alpha^{\pm}(\varphi_i^-) , \\ c_{12}^{\pm}&=\mp i\frac{\xi'}{\xi}\cos(\varphi_t)e^{\pm i\delta_i^+}\alpha^{\pm}(\varphi_i^+) , \\ c_{21}^{\pm}&=\mp i\frac{\xi}{\xi'}\cos(\varphi_t)e^{\pm i\delta_i^-}\beta^{\pm}(\varphi_i^-) , \\ c_{22}^{\pm}&=\pm\frac{1}{\xi'}\cos(\varphi_t)e^{\pm i\delta_i^+}\beta^{\pm}(\varphi_i^+) . \end{aligned} \tag{27}$$

Then, eliminating $\hat{T}_E$ and $\hat{T}_H$ from (25a) and (25b) gives

$$\begin{aligned} c_H^{+}\hat{Q}_H+c_{12}^{+}\hat{Q}_E&=c_{11}^{-}\hat{R}_H+c_{12}^{-}\hat{R}_E , \\ c_{21}^{+}\hat{Q}_H+c_{22}^{+}\hat{Q}_E&=c_{21}^{-}\hat{R}_H+c_{22}^{-}\hat{R}_E . \end{aligned} \tag{28}$$

The solution of the system (28) is

$$\begin{aligned} \hat{R}_H&=r_{11}\hat{Q}_H+r_{12}\hat{Q}_E , \\ \hat{R}_E&=r_{21}\hat{Q}_H+r_{22}\hat{Q}_E , \end{aligned} \tag{29}$$

with

$$\begin{aligned} r_{11}&=\Gamma^{-1}(c_{22}^{-}c_{11}^{+}-c_{12}^{-}c_{21}^{+}) , \quad r_{12}=\Gamma^{-1}(c_{22}^{-}c_{12}^{+}-c_{12}^{-}c_{22}^{+}) , \\ r_{21}&=\Gamma^{-1}(c_{11}^{-}c_{21}^{+}-c_{21}^{-}c_{11}^{+}) , \quad r_{22}=\Gamma^{-1}(c_{11}^{-}c_{22}^{+}-c_{21}^{-}c_{12}^{+}) , \\ \Gamma&=c_{11}^{-}c_{22}^{-}-c_{12}^{-}c_{21}^{-} . \end{aligned} \tag{29'}$$

Now substituting (28) into (25a) gives

$$\begin{aligned} \hat{T}_H&=t_{11}\hat{Q}_H+t_{12}\hat{Q}_E , \\ \hat{T}_E&=t_{21}\hat{Q}_H+t_{22}\hat{Q}_E , \end{aligned} \tag{30}$$

with

$$\begin{aligned} t_{11}&=(1+r_{11}-i\xi^{-1}r_{21})e^{i(\delta_i^- -\delta_t)} , \\ t_{12}&=-i\xi^{-1}(1+r_{22}+i\xi r_{12})e^{i(\delta_i^+ -\delta_t)} , \\ t_{21}&=-i\xi(1+r_{11}+i\xi^{-1}r_{21})e^{i(\delta_i^- -\delta_t)} , \\ t_{22}&=(1+r_{22}-i\xi r_{12})e^{i(\delta_i^+ -\delta_t)} . \end{aligned} \tag{30'}$$

Relations (29) and (30) are the Fresnel formulas for the reflected and refracted electromagnetic fields at a chiral-achiral interface.

III. ELECTROMAGNETIC WAVES THROUGH A CHIRAL SLAB

Let us now consider a chiral slab of thickness d immersed in an achiral medium and confined between the two planes $x=0$ and $x=d$. A linearly polarized wave is incident at an angle θ_i on the interface $x=0$. We assume [4] that four waves propagate in the slab: two toward the interface $x=d$ and the other two toward the interface $x=0$. Then, one has 11 angles of incidence, reflection, and refraction as shown in Fig. 1,

$$\theta_i,\theta_r,\theta_t^{\pm},\varphi_i^{\pm},\varphi_r^{\pm},\varphi_t,\chi_i^{\pm} , \tag{31}$$

where $\chi_i^{\pm}$ is the angle of incidence at $x=0$ for the wave $\hat{R}$ reflected at $x=d$. But they are not independent since one has

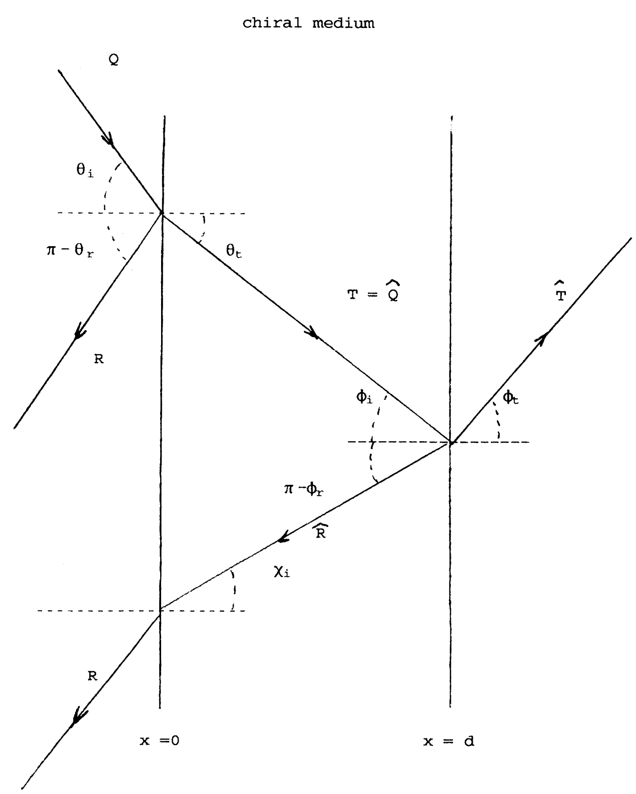

FIG. 1. A homogeneous isotropic chiral slab immersed in a homogeneous isotropic achiral medium. The notation shown refers to a plane wave incident from the left.

$$\varphi_i^{\pm}=\theta_t^{\pm}\ , \quad \chi_i^{\pm}=\pi-\varphi_r^{\pm}\ . \tag{31'}$$

Now the Descartes-Snell relations are

$$n'\sin\theta_i=n'\sin\theta_r=n^{\pm}\sin\theta_t^{\pm}=n^{\pm}\sin\varphi_r^{\pm}=n'\sin\varphi_t\ , \tag{32}$$

which implies $\theta_i=\varphi_t$ in addition to $\theta_r=\pi-\theta_i$, $\varphi_r^{\pm}=\pi-\theta_t^{\pm}$.

The incident, reflected, and refracted fields are

$$\begin{aligned} &Q=(Q_H,Q_E)\ , \quad R=(R_H,R_E)\ ,\\ &T=(T_H,T_E)=\hat{Q}=(\hat{Q}_H,\hat{Q}_E)\ ,\\ &\hat{R}=(\hat{R}_H,\hat{R}_E)\ , \quad \hat{T}=(\hat{T}_H,\hat{T}_E)\ . \end{aligned} \tag{33}$$

Let us now apply the boundary conditions (15) at $x=0$. The relations (17a) are still valid while instead of (17b) one has

$$\begin{aligned} &\Lambda_y^+=2\sqrt{\epsilon}(T_E+\hat{R}_E)\ ,\\ &\Lambda_y^-=-2i\sqrt{\mu}(T_H+\hat{R}_H)\ ,\\ &\Lambda_z^+=2i\cos(\theta_t^+)\sqrt{\epsilon}(T_E-\hat{R}_E)\ ,\\ &\Lambda_z^-=-2\cos(\theta_t^-)\sqrt{\mu}(T_H-\hat{R}_H)\ , \end{aligned} \tag{34}$$

where we used the relation $\varphi_r^{\pm}=\pi-\theta_t^{\pm}$.

From (17a) and (34) we get the four equations

$$\begin{aligned} &Q_H+R_H=T_H+\hat{R}_H-i\xi^{-1}(T_E+\hat{R}_E)\ ,\\ &Q_E+R_E=T_E+\hat{R}_E-i\xi(T_H+\hat{R}_H) \end{aligned} \tag{35a}$$

and

$$\begin{aligned} \cos(\theta_i)\xi'(Q_H-R_H)&=\cos(\theta_t^-)\xi(T_H-\hat{R}_H)\\ &\quad -i\cos(\theta_t^+)(T_E-\hat{R}_E)\ ,\\ \cos(\theta_i)\xi'^{-1}(Q_E-R_E)&=\cos(\theta_t^+)\xi^{-1}(T_E-\hat{R}_E)\\ &\quad -i\cos(\theta_t^-)(T_H-\hat{R}_H)\ . \end{aligned} \tag{35b}$$

Let us remark that in (17a) the amplitudes (R_E,R_H) represent the reflected field due to the incident field (Q_E,Q_H). But in (35) (R_E,R_H) include also the refracted amplitudes due to the field $(\hat{R}_E,\hat{R}_H)$ reflected from the interface $x=d$.

Eliminating R_E and R_H from relations (35) gives

$$Q=\mathcal{A}^+T+\mathcal{A}^-R\ , \tag{36}$$

where $\mathcal{A}^+$ is the matrix with elements (20a) that from now on we denote $a_{rs}^+, r,s=1,2$ and $\mathcal{A}^-$ the matrix with elements a_{rs}^- obtained by changing $(1+\cdots)$ into $(1-\cdots)$ in (20a).

We now use relations (29) and (30) supplied by the boundary conditions at the interface $x=d$. Since according to (33) one has $(T_E,T_H)=(\hat{Q}_E,\hat{Q}_H)$ we may write (29)

$$\hat{R}=\mathcal{R}T\ , \tag{37}$$

where $\mathcal{R}$ is the matrix with elements (29′).

Eliminating $\hat{R}_E$ and $\hat{R}_H$ from (36) and (37) gives

$$Q=\mathcal{U}T\ , \tag{38}$$

where $\mathcal{U}$ is the matrix with elements [using (20a) and (29′)]

$$\begin{aligned} &u_{11}=a_{11}^++r_{11}a_{11}^-+r_{12}a_{12}^-\ , \quad u_{12}=a_{12}^++r_{12}a_{11}^-+r_{22}a_{12}^-\ ,\\ &u_{21}=a_{21}^++r_{11}a_{12}^-+r_{21}a_{22}^-\ , \quad u_{22}=a_{22}^++r_{12}a_{21}^-+r_{22}a_{22}^-\ . \end{aligned} \tag{38'}$$

The solution of system (38) is

$$T=\mathcal{U}^{-1}Q\ . \tag{39}$$

Now from (35a) and (37) we get

$$R=-Q+\mathcal{V}T\ , \tag{40}$$

with

$$\begin{aligned} &v_{11}=1+r_{11}-i\xi^{-1}r_{21}\ , \quad v_{12}=-i\xi^{-1}(1+r_{22}+i\xi r_{12})\ ,\\ &v_{21}=-i\xi(1+r_{11}+i\xi^{-1}r_{21})\ , \quad v_{22}=1+r_{22}-i\xi r_{12}\ . \end{aligned} \tag{40'}$$

Substituting (39) into (40) gives the amplitude of the electromagnetic field reflected by the achiral slab

$$R=-(1-\mathcal{U}^{-1}\mathcal{V})Q\ . \tag{41}$$

To obtain the amplitudes refracted by the slab we use once more the equality $(\hat{Q}_E,\hat{Q}_H)=(T_E,T_H)$ and we write (30)

$$\hat{T}=\mathcal{T}T\ , \tag{42}$$

where $\mathcal{T}$ is the matrix with elements (30′).

Substituting (39) into (42) gives

$$\hat{T}=\mathcal{T}\mathcal{U}^{-1}Q\ . \tag{43}$$

The expressions (43) represent the field diffracted by a chiral slab.

To sum up, in terms of the incident time-harmonic linearly polarized plane wave $Q=\binom{Q_H}{Q_E}$ the expressions of the reflected field $R=\binom{R_H}{R_E}$ at the interface $x=0$ and of the refracted field $\hat{T}=\binom{\hat{T}_H}{\hat{T}_E}$ at the interface $x=d$ of a chiral slab are

$$R=-(1-\mathcal{U}^{-1}\mathcal{V})Q\ , \quad \hat{T}=\mathcal{T}\mathcal{U}^{-1}Q\ , \tag{44}$$

where $\mathcal{U}$, $\mathcal{V}$, and $\mathcal{T}$ are the matrices (38′), (40′), and (30′), respectively.

IV. CONCLUSIONS

In this work we used the 3D complex formalism to discuss the propagation of time-harmonic plane waves in a chiral slab assuming that four waves propagate in the slab. We do not discuss the physical implications of relations (44) since it was already made, for instance, in [4].

The real question is whether the 3D complex formalism is simpler than the Maxwell-Heaviside formalism to discuss electromagnetic wave propagation in chiral media. Since the 3D complex formalism is not covariant under the space inversions the answer is *a priori* yes for people thinking that a physical problem is easier to solve when the mathematical formalism respects the physical symmetries.

This *a priori* positive answer is justified by the comparison of the results obtained here with those of [4] and in fact we used the same assumptions as in [4] to make this comparison possible. According to (44) the reflected and refracted amplitudes by the chiral slab require only the inversion of the 2×2 matrix $\mathcal{U}$ while in [4] one has to inverse a 8×8 matrix a much more difficult task.

The author regrets having no access to a computer, making him unable to discuss more realistic problems, but he hopes that some people will be convinced that the 3D complex formalism is worth using.

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[1] P. Hillion, Phys. Rev. E **47**, 1365 (1993).
[2] E. J. Post, *Formal Structure of Electromagnetics* (North-Holland, Amsterdam, 1962).
[3] M. P. Silvermann, J. Opt. Soc. Am. A **3**, 830 (1986).
[4] S. Bassiri, C. H. Pappas, and N. Engheta, J. Opt. Soc. Am. A **5**, 1450 (1988).
[5] A. Lakhtakia, V. V. Varadan, and V. K. Varadan J. Opt. Soc. Am. A **7**, 1654 (1990).